Classical solids dynamics as 4D statics of elastic strings

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Abstract

Variational principle for a solid in classical mechanics is formulated in terms of a thin elastic 4D bar strain in events space $\mathbb{M}_4$ of special relativity. It is shown, that the sum of elastic 4-energies of weak twist and bending under some identifications takes the form of classical non-relativistic action for a solids dynamics. The necessary conditions on 4D bar parameters and elastic constants, providing validity of Newton mechanics, are found.

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1 INTRODUCTION

In recent years extended objects with small effective dimensions — strings and membranes — hold central position in modern theoretical physics [1]-[7]. Due to a rich physical and geometrical content, they are, probably, the most suitable candidates for unified models of space-time, particles and fields. Apart from the unification ideas (or partly due to ones), the string and brane concepts suggest that there are many physical topics, considered earlier as quite different, which have deep interrelations between each other. An example is intriguing physical analogy between elasticity theory [8] and Einstein gravity [9], originally noted by Born [10]. Based on this analogy new approach to a space-time-matter dynamics has been developed in works [11, 12, 13, 14]. It has required some reformulations of the both theories: elasticity theory has been generalized to multidimensional elastic bodies and Einstein gravity (Einstein-Gilbert action for gravitational field) has been written in terms of isometric embedding [15]. Investigation of the analogy within the context of the approach reveals very simple physical interpreting of GR action: *total action*

$$\mathfrak{s} = \mathfrak{s}_{\mathrm{g}} + \mathfrak{s}_{\mathrm{m}} \tag{1}$$

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for a system "gravitational field + matter" has mechanical sense of multidimensional elastic energy of a strained thin 4D plate:

$$\mathfrak{F} = \mathfrak{F}_{\mathrm{b}} + \mathfrak{F}_{\mathrm{s}}, \tag{2}$$

where the first term is responsible for its bending, second — for tangent to its surface stretches-shears. The term "4D plate"[1] means multidimensional body, whose sizes along some 4D Riemannian manifold $\mathbb{V}_4$ are much more than orthogonal to $\mathbb{V}_4$ ones. Status analysis of a number of objects within classical field theory (lagrangian, metrics and metric energy-momentum tensor, space-time signature, boundary conditions) has been made in frame of the approach in [13]. Dimensional analysis gives the following relation between Einstein constant $\varkappa$ and elastic constants of the plate — Young modulus and thicknesses:

$$\frac{1}{\varkappa} \sim Eh^{N+3},$$

where N — number of extradimensions. It supports old Sacharov's idea about elastic origin of Einstein constant $\varkappa$ [16]. Embedding theory and elasticity theory have been applied earlier independently in a number of works on gravity, brane physics and quantum theory [17] (and ref-s therein), [18, 19, 20].

If the discussed analogy is not occasional, then multidimensional elasticity should be relevant also to clearing of physical picture in case of simpler systems. In present article we apply the approach to solids dynamics in classical mechanics. In other words, our aim is derivation of mechanical action from the multidimensional elastic energy functional. Our consideration will naturally lead us again to the 4D objects with small (equal to 1) effective dimension — *thin bars*, posed in 4D Minkowski space, which from now on will be shortly referred to as *4-bars.* These objects are of different nature, than ordinary classical strings, considered within the field theory. Following analysis shows, that the 4-bars deformation theory reproduces classical Newton mechanics, when the bars posses a very strong tension. By standard terminology of elasticity theory, we call such strongly tensed bars *strings.* Its difference from the ordinary ones is that they are time-like and correspond to world tubes of finite space-like thickness within special relativity (SR), while classical string is a space-like 1D object (i.e. has nonzero length and zero thickness)[2]. On the other hand, in contrast to world tubes of SR, the strings studied here possess 4D elasticity, in particular, time-like one. To distinguish such strings from standard ones,

[1]Here and below we distinguish strongly tensed objects (strings and membranes) from a more general (bars and plates) arbitrarily stressed ones.

[2]Though moving ordinary string generates two-dimensional matter sheet in $\mathbb{M}_4$, it possess a null thickness and so should be related to a more particular subclass of a bars: tubes (closed string) or bands (open string).

we'll shortly call them through the paper *4-strings,* reminding that we deal with 4D object. Note, that our approach overlaps with Pavšič works [3, 4] in two points: we use the similar concept of observer (simultaneity hypersurface and its motion in $\mathbb{M}_4$) and get observable 3-D classical equations of motions as consequence of a more general multidimensional physics with "frozen" matter.

The paper is organized as follows. In Sec.2 we give sketch of events space properties and 4D kinematics within SR. Here we also introduce the definition of general 4-body by analogy with 3D case.

Sec.3 restricts our attention to the important subclass of 4-bodies — thin 4-bars, which are 4D "pre-images" of observable 3-bodies in classical mechanics.

Sec.4 is devoted to a 4D generalization of standard elasticity theory.

In Sec.5 we derive elastic 4-energy functional for the case of pure twist of a thin 4-bar. We show, that the 4-energy goes into classical rotational part of mechanical action, when 4-bars space sections remain unstrained (rigid 3-body) and the twist is weak (non-relativistic rotation).

Weak bending of a 4-bar is investigated in Sec.6. We give general relativistic description of 4-bar stretchless strain, including both bending and twist and then extract the part, which is responsible for pure bending. It turns out, that the elastic energy of bending is quadratically depends on acceleration of 3-body, so the equation of motion of 3-bodies (static equilibrium of 4D ones) should have a forth order. To reject this contradiction to experience, we go from 4-bars to 4-strings and require negligibility of its bending energy in comparison with stretching one. It gives us additional restriction, clearing 4-string physics (see noneq.(36) — strong tension condition) and right action of classical mechanics — quadratic over velocities in nonrelativistic case.

Sec.7 contains qualitative discussion of classical mechanics within the frame of developed approach.

Throughout the whole paper, unless otherwise specified, standard indexless notations are used. Particularly, small Latin, Greek or Gothic letters denote 3D objects, capital ones — corresponding 4D ones.

2 SOLIDS KINEMATICS IN SPECIAL RELATIVITY

The aim of this section is to remind mathematical properties of events space of SR, which lies in foundation of 4D bar mechanics. SR endows affine space of events $\mathbb{A}_4$ by the *Lorentzian structure*, which contains *pseudoeuclidian structure* and *relativistic symmetry group.* The first defines distance S_{AB} between any two points-events

$A, B \in \mathbb{A}_4$:

$$S_{AB} = S_{BA} = \sqrt{(\boldsymbol{V}_{AB}, \boldsymbol{V}_{AB})},$$

where $\boldsymbol{V}_{AB}$ — 4-vector, (element of affine vector space $\mathbb{W}_4$) and scalar product is determined by *Minkowski metrics* with signature $(+,-,-,-)$. The hyperbolicity of Minkowski metric leads to the three type of vectors in $\mathbb{W}_4$: *time-like* $((\boldsymbol{V}, \boldsymbol{V}) > 0)$, *space-like* $((\boldsymbol{V}, \boldsymbol{V}) < 0)$ and *isotropic* $((\boldsymbol{V}, \boldsymbol{V}) = 0)$.

The second — *Poincare group* P — is subgroup of a group of general non-homogeneous linear transformations $\mathrm{GL}(4, \mathrm{R})$ in $\mathbb{A}_4$, which don't change Minkowski metrics. Space of events with Minkowski metric and isometry Poincare group is called *space-time* $\mathbb{M}_4$.

Third element, we need to introduce, is related to an observable in $\mathbb{M}_4$ physics. Let $A, B \in \mathbb{M}_4$ — pair of events, connected by a simple smooth path γ_{AB}. Then time interval T_{AB} between A and B along γ_{AB} is the integral

$$\int\limits_{\gamma_{AB}} \tilde{\boldsymbol{T}},$$

where $\tilde{\boldsymbol{T}} = \tilde{\boldsymbol{T}}_\mu dX^\mu$ — *relativistic form of time*, also referred to in papers on the subject as τ-field or reference frame form [21]. In a difference with non-relativistic mechanics, where form $\tilde{\boldsymbol{t}}$ is unique (absolute) for all reference frame, number forms of time in $\mathbb{M}_4$ is infinitely large. Note, that absence of the form $\tilde{\boldsymbol{T}}$ within the SR, makes it physically contentless, since we lose any possibility to compare the theory with experiment[3]. The only algebraic condition imposed on $\tilde{\boldsymbol{T}}$ is

$$(\tilde{\boldsymbol{T}}, \tilde{\boldsymbol{T}}) = 1,$$

which means, that $\tilde{\boldsymbol{T}}$ defines on a whole $\mathbb{M}_4$ unit time measure. Decomposition of 4D objects onto its time-like and 3D space-like projections can be carried out with the help of *projection operators*. They are constructed through the all possible tensor powers of $\tilde{\boldsymbol{T}}$ and of metrics of local space-like section $\boldsymbol{H}$, defined by equation:

$$(\ ,\ ) = \tilde{\boldsymbol{T}} \otimes \tilde{\boldsymbol{T}} - \boldsymbol{H}.$$

Particular interest has the case, when the form $\tilde{\boldsymbol{T}}$ admits global decomposition of $\mathbb{M}_4$ onto time and space. Necessary and sufficient condition for the decomposition is *integrability of the form* $\tilde{\boldsymbol{T}}$:

$$\boldsymbol{d}\tilde{\boldsymbol{T}} \wedge \tilde{\boldsymbol{T}} = 0,$$

where $\boldsymbol{d}$ — external differential. If it is valid, then differential equation $\tilde{\boldsymbol{T}} = 0$ has a solution $v(X) = t = \text{const}$, defining space-like hypersurface of simultaneousity

[3] So we follow to the Arnold's book [22], where similar three blocks of classical Newton mechanics — Galilean structure, Galilean symmetry group and form of time — are accented.

$\mathbb{V}_3^t$, to which coordinate time t corresponds. This time is determined up to an arbitrary monotonic function $\varphi: \ t' = \varphi(t)$. At the surface $\mathbb{V}_3^t$, which is, generally speaking, Riemannian manifold with metrics $-\boldsymbol{H}$, any 3D coordinates and their points transformations $x' = x'(x)$ are admissible.

By analogy to bodies of classical mechanics, let us call continuous non-negative real function ϱ: $\mathbb{M}_4 \to \mathbb{R}$ *4-density of a mass.* Then *4-body* will be connected (in affine topology of $\mathbb{A}_4$) closed set $V \in \mathbb{M}_4$, for which $\varrho \neq 0$. *4-volume* of a 4-body is the integral:

$$\mathcal{V} = \int\limits_V 1,$$

and *4-mass* of the body — integral with the weight ϱ:

$$\mathcal{M} = \int\limits_V \varrho.$$

The both integrations are carried out with standard measure on $\mathbb{M}_4$:

$$d\mathrm{vol}_4 = \sqrt{-\Delta} dX^0 \wedge dX^1 \wedge dX^2 \wedge dX^3,$$

where Δ — determinant of Minkowski metrics. It will become clear from the following consideration, that in a static world of 4-bars, 4-mass doesn't describe their inertial properties, but should be related to the elastic ones.

Having integrable $\tilde{\boldsymbol{T}}$-form and 4-body, one can consider usual *3-bodies* as closed sets $V \cap \mathbb{V}_3^t = \cup v_i^t$, which give non-vanishing restrictions: $\varrho|_{\mathbb{V}_3^t} \equiv \rho^t(x) > 0$. It is natural to call the restriction ρ^t *3-density of a mass.* So defined, 3-densities and 3-bodies correspond to ones in classical Newton mechanics, with the difference, that the first are depend on a choice of $\tilde{\boldsymbol{T}}$-form. To be short we'll not show this dependence explicitly. Using decomposition of the form of 4-volume: $d\mathrm{vol}_4 = \tilde{\boldsymbol{T}} \wedge \mathrm{vol}_3^{\mathrm{t}}$, one can calculate integral values of *3-volume* and *3-mass*:

$$\nu(t) = \int\limits_{v^t} 1; \quad m(t) = \int\limits_{v^t} \rho^t(x). \tag{3}$$

From the 4D viewpoint, bodies, considered as different in $\mathbb{V}_3^t$ (i.e. satisfying $v_i^t \cap v_j^t = \varnothing$), can be different sections of the same 4-body in $\mathbb{M}_4$ (if in the past or future they satisfy condition $v_i^{t'} \cap v_j^{t'} \neq \varnothing$).

Mapping of the pair — form of time and 4-body — into set of 3-bodies: $\{\tilde{\boldsymbol{T}}, V\} \to \{v^t \in \mathbb{V}_3^t\}$ induces mapping, $t \in \mathbb{R} \to v^t$, which we call *motion* in $\mathbb{V}_3^t$. It is important to note, that this motion is illusion from the $\mathbb{M}_4$ viewpoint: it is sequence of intersection traces of $\mathbb{V}_3^t$ with immobile in $\mathbb{M}_4$ 4-body[4]. One can say, that the set of 4-bodies in $\mathbb{M}_4$ is "frozen" history. Fragment of 4-body is shown in Fig.1.

[4] Pavšič calls it in [4] Σ-motion (in our notation $\Sigma = \mathbb{V}_3$)

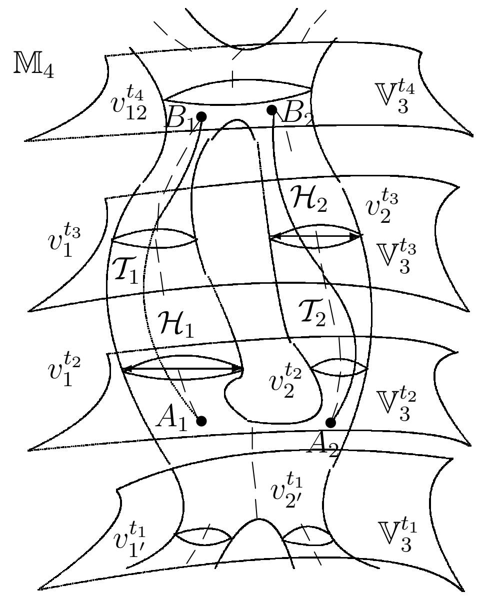

Figure 1: Fragment of a 4-body. Corresponding 3-bodies v_i^t are shown in some reference frame. Time-like and space-like sizes of 4-bars (see following section) are equal to $\mathcal{T}_1$, $\mathcal{T}_2$, $\mathcal{H}_1$, $\mathcal{H}_2$ correspondingly.

3 THIN 4-BARS

Theoretically, one can consider 4-bodies, having restricted over t support in some reference frame $\tilde{\boldsymbol{T}}$:

$$\operatorname*{supp}_{t\in\mathbb{R}} \varrho(t,x) = T \subset \mathbb{R}.$$

From the viewpoint of $\mathbb{V}_3$ these 4-bodies may be looked at as suddenly appearing and disappearing 3-bodies. Such behaviour of classical (non-quantum) objects is in drastic contradiction with observable continuous dynamics of any 3-bodies in $\mathbb{V}_3$. So, to avoid paradoxes and violation of classical conservation laws (for example, energy), one can put

$$\operatorname*{supp}_{t\in\mathbb{R}} \varrho(t,x) = \mathbb{R},$$

that implies eternity of mechanical Universe. However, cosmological reasons demand and our everyday experience admits weaker condition on the support ϱ in $\mathbb{M}_4$. Lets call *space-like size* of a 4-body

$$\mathcal{H} \equiv \max_{t;\ A,B\in v^t} \left| \int\limits_{\Gamma^-_{AB}\subset v^t} \sqrt{\boldsymbol{H}(d\boldsymbol{x}, d\boldsymbol{x})} \right|,$$

where Γ^-_{AB} — space-like geodesic, connecting some two points A, B of 3-body v^t. Roughly speaking, space-like size $\mathcal{H}$ is maximal space-like size, chosen from all space sections of the 4-body (see Fig.1). By analogy, *time-like size* of the 4-body will be

called

$$\mathcal{T} \equiv \max_{A,B\in V} \int\limits_{\Gamma^+_{AB}} \tilde{\boldsymbol{T}},$$

where Γ^+_{AB} — time-like geodesic, connecting some points A and B of the 4-body (see Fig.1). Now we can formulate condition, which is necessary for agreement of properties of 4-bodies world with observable 3-bodies ones: *since observable classical 3-bodies easily show its bounds in 3-space, but don't show bounds in time (i.e. don't appear out of nothing and disappear in to nothing), then for every real 4-body we should put*

$$\mathcal{T} \gg \mathcal{H}. \tag{4}$$

Note, that inspite of dependency of $\mathcal{H}$ and $\mathcal{T}$ on reference frame, the non-equality is relativistic invariant, by the well known properties of Lorentz transformations. Such 4-bodies, in terms of elasticity theory can be called *thin 4-bars*.

4 4-D ELASTICITY THEORY

Let's call *strain of 4-body* the diffeomorphism

$$V \to V_{\boldsymbol{\Xi}}, \tag{5}$$

given by the smooth [5] *displacement vector field* $\boldsymbol{\Xi} = \boldsymbol{\Xi}(X),\ \{X\} \in V$. As usually we define *strain tensor* $\boldsymbol{D}$:

$$dS^2_{\boldsymbol{\Xi}} = dS^2 + 2\boldsymbol{D}(dX, dX),$$

where

$$\boldsymbol{D} = \frac{1}{2}(\overrightarrow{\partial} \otimes \boldsymbol{\Xi} + \boldsymbol{\Xi} \otimes \overleftarrow{\partial}) + \frac{1}{2}(\partial' \otimes \partial'')(\boldsymbol{\Xi}', \boldsymbol{\Xi}''), \tag{6}$$

where ∂ — tensor partial derivative operators, arrows and primes denote their action on functions. In standard 3D linear elasticity theory second term in (6) is commonly omitted, assuming small value of relative strains: $\max|\partial\xi| \ll 1$, for which Hooks law is valid[6]. This condition is invariant under transformation of Galilean group acting in events space of classical mechanics by the compactness of its subgroup SO(3). The similar form of 4D bodies strains small value conditions:

$$\max|\partial\boldsymbol{\Xi}| \ll 1, \tag{7}$$

is, generally speaking, invalid, since the group $SO(1,3)$ is — noncompact and pseudorotations of reference frame can violate it. However, since we only try to formulate

[5] Components $\boldsymbol{\Xi} \in C^k,\ k \geq 4$.

[6] This is impossible, for example, for weak bending of a thin plates, when the first linear over derivatives of displacement vector term is generally absent.

lagrangian formalism of classical non-relativistic mechanics in terms of elasticity theory, we can restrict our attention to pseudorotations near group unity. Such transformations, which are in fact translations in space of velocities, leave the condition (7) invariant.

Decomposition of a *4-density of elastic free energy* in linear elasticity theory contains only quadratic over components $\boldsymbol{D}$ terms:

$$F = \zeta \boldsymbol{D}^2 + \frac{\lambda}{2} \left[\mathrm{Tr}\, \boldsymbol{D}\right]^2 , \tag{8}$$

where ζ, λ — phenomenological elastic *Lame coefficients*. Stress 3-form $*\boldsymbol{\sigma}$, defining measure $d\tilde{\boldsymbol{F}}$ of *elastic 4-force 1-form*[7]

$$\tilde{\boldsymbol{F}}(U) \equiv \int\limits_{\partial U} *\tilde{\boldsymbol{\sigma}} = \int\limits_{U \subset V} \boldsymbol{d} * \tilde{\boldsymbol{\sigma}} \equiv \int\limits_{U \subset V} d\tilde{\boldsymbol{F}},$$

where $*$ — dual conjugation in $\mathbb{M}_4$, U — arbitrary region inside 4-body V, can be calculated by the formula [8]:

$$\boldsymbol{\sigma} = \frac{\partial F}{\partial \boldsymbol{D}}. \tag{9}$$

Boundary conditions at a surface of the 4-body are:

$$\boldsymbol{\sigma}|_{\partial V} = \tilde{\boldsymbol{P}}, \tag{10}$$

where $\tilde{\boldsymbol{P}}$ — surface density of external 4-forces. Equation (10) implicitly assumes validity of third Newton law for the bodies in $\mathbb{M}_4$, which, as it will be seen from the following consideration, is the only mechanical law for 4-bars. This one, as it has been noted in [23], differing from the other two Newton laws, implies very general assumptions about force function: it should only possess an additivity property on the bodies of Universe: $\tilde{\boldsymbol{F}}(U_1 \cup U_2) = \tilde{\boldsymbol{F}}(U_1) + \tilde{\boldsymbol{F}}(U_2)$ for any U_1, U_2 satisfying $U_1 \cap U_2 = \varnothing$.

5 TWIST STRAINING OF A 4-BAR

Let's consider a thin bar, occupying a region $V \in \mathbb{M}_4$ in unstrained case, given in Cartesian coordinates $\{X\}$ by the following way:

$$\mathrm{supp}\, \varrho = V = v \times [X^0_1; X^0_2],$$

— where v — 3D set lying inside 2-surface, defined by some equation $\Phi(x) = 0$. Let's take form of time $\tilde{\boldsymbol{T}} = dX^0$, then $\mathbb{V}^t_3 = \mathbb{E}^t_3$, $v^t = V \cap \mathbb{E}^t_3 = v = \mathrm{const}$, and equation $\Phi(x) = 0$ describes 3-space form of some 3-body, observed in reference

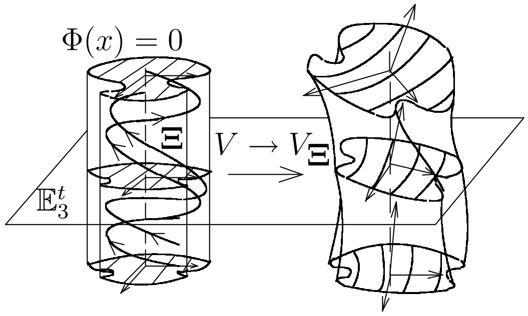

Figure 2: Twist strain of 4-bar. Coordinate system and form of time are consistent with each other. In unstrained 4-bar state (from the left), 3-body is in a rest. After straining (from the right) 3-body is twisted and curved, while inertia line remains rectilinear.

frame $\tilde{\boldsymbol{T}}$ in a rest (see Fig.2). By a parallel transition in $\mathbb{E}_3$, one can put origin of space part of coordinate system at the mass center $\boldsymbol{x}_0$ of the 3-body.

Let's call *twist strain* the special kind of diffeomorphism (5), given by the following displacement vector field:

$$\boldsymbol{\Xi}(X) = \boldsymbol{\Xi}(t, \boldsymbol{x}) = (\Xi_0, \hat{\mathrm{O}}\boldsymbol{x}),$$

where $\hat{\mathrm{O}}$ — t-dependent 3D orthogonal matrix, defining twist angle of 3-body of the 4-bar at arbitrary moment t. Its time derivative can be expressed through the *twist 3-vector* $\boldsymbol{\omega}$ by the following expression:

$$\frac{d\hat{O}}{dt} = [\boldsymbol{\omega},\ ],$$

where $[,]$ — ordinary 3D vector product. Within standard elasticity theory it has the meaning of twist angle of a bar per unit of its length, while in 4-bars physics $\boldsymbol{\omega}(t)$ is, in fact, angular velocity of 3-body, induced by the 4-bar twist strain [8]. The component $\Xi_0 = \Xi_0(\boldsymbol{\omega}, t, \boldsymbol{x})$.

Weak twist condition

$$\omega\mathcal{H} \ll 1,$$

which we postulate now, and which means in terms of SR slow (non-relativistic) rotation, let's expand Ξ_0 in the row over powers of $\boldsymbol{\omega}$:

$$\Xi_0 = \boldsymbol{\psi}_1(\boldsymbol{\omega}) + \boldsymbol{\psi}_2(\boldsymbol{\omega}, \boldsymbol{\omega}) + \boldsymbol{\psi}_3(\boldsymbol{\omega}, \boldsymbol{\omega}, \boldsymbol{\omega}) + \dots,$$

where

$$\boldsymbol{\psi}_n = \frac{\partial^n_{\boldsymbol{\omega}} \Xi_0}{n!}\,|_{\boldsymbol{\omega}=0}$$

[7]We use notation $*\boldsymbol{\sigma}$ instead of $\boldsymbol{\sigma}$ to obtain standard relation (9). In coordinate form $*\boldsymbol{\sigma}_{\alpha\beta\gamma\delta} = \boldsymbol{E}_{\rho\beta\gamma\delta}\boldsymbol{\sigma}^{\rho}_{\alpha}$, where $\boldsymbol{E}$ — absolute antisymmetric Levi-Civita tensor in $\mathbb{M}_4$.

[8]For convenience we include light velocity c into unit measure of time in all 4D expressions and return it to apparent kind in all final results for comparing with classical dynamics

— *twist functionals* and lets then restrict our consideration by the first term of the decomposition (which we'll notate without index "1"). Note, that twist functionals describe space deformation of 3D sections (3-bodies) of the 4-bar and their displacement along t-axe.

Non-zero components of linearized strain tensor (6) have the form:

$$\boldsymbol{D}_{00} = (\dot{\boldsymbol{\psi}}, \boldsymbol{\omega}) + (\boldsymbol{\psi}, \dot{\boldsymbol{\omega}}); \quad \boldsymbol{D}_{0i} = \frac{1}{2}((\boldsymbol{\omega}, \partial_i \boldsymbol{\psi}) + [\boldsymbol{\omega}, \boldsymbol{x}]_i), \tag{11}$$

where dot denotes time derivative, $\partial_i \equiv \partial/\partial x^i$. Energy density (8) of twist strain then takes the form:

$$F_{\rm tw} = (\zeta + \frac{\lambda}{2})\boldsymbol{D}_{00}^2 - \zeta \boldsymbol{D}_{0i}^2 = \frac{\boldsymbol{J}_1(\boldsymbol{\omega}, \boldsymbol{\omega})}{2} + \boldsymbol{J}_2(\boldsymbol{\omega}, \dot{\boldsymbol{\omega}}) + \frac{\boldsymbol{J}_3(\dot{\boldsymbol{\omega}}, \dot{\boldsymbol{\omega}})}{2}, \tag{12}$$

where tensors $\boldsymbol{J}_1, \boldsymbol{J}_2, \boldsymbol{J}_3$ are defined by the expressions:

$$\boldsymbol{J}_1 = 2(\zeta + \frac{\lambda}{2})\dot{\boldsymbol{\psi}} \otimes \dot{\boldsymbol{\psi}} - \frac{\zeta}{2}((\partial \boldsymbol{\psi}, \otimes \partial)_3 \boldsymbol{\psi} + [\boldsymbol{x}, \overrightarrow{\partial}] \dot{\otimes} \boldsymbol{\psi} + \boldsymbol{\psi} \otimes [\boldsymbol{x}, \overleftarrow{\partial}] + \boldsymbol{x}^2(\ ,\ )_3 - \boldsymbol{x} \otimes \boldsymbol{x});$$

$$\boldsymbol{J}_2 = 2(\zeta + \frac{\lambda}{2})\dot{\boldsymbol{\psi}} \otimes \boldsymbol{\psi}; \quad \boldsymbol{J}_3 = 2(\zeta + \frac{\lambda}{2})\boldsymbol{\psi} \otimes \boldsymbol{\psi}.$$

Here $(\ ,\ )_3 \equiv \boldsymbol{H}$ in our fixed reference frame. From the viewpoint of classical mechanics the bilinear forms $\boldsymbol{J}_2(\boldsymbol{\omega}, \dot{\boldsymbol{\omega}})$, $\boldsymbol{J}_3(\dot{\boldsymbol{\omega}}, \dot{\boldsymbol{\omega}})$, and also the first term in $\boldsymbol{J}_1(\boldsymbol{\omega}, \boldsymbol{\omega})$ describe relativistic corrections to the kinetic rotational energy of a 3D solid; three addends in $\boldsymbol{J}_1$, including space derivatives of $\boldsymbol{\psi}$ describe ordinary elastic strain of 3-body, generated by rotation, and two remaining addendums in $\boldsymbol{J}_1$ determine kinetic energy density of absolutely rigid 3-body. Really, for such body $\boldsymbol{\psi} \equiv 0$ and we get:

$$\mathfrak{F}_{\rm tw}|_{\boldsymbol{\psi}\equiv 0} = \int\limits_V F_{\rm tw}|_{\boldsymbol{\psi}\equiv 0}\, dt \wedge d{\rm vol}_3 = -\int\limits_V \frac{\zeta}{2}\boldsymbol{J}_1(\boldsymbol{\omega}, \boldsymbol{\omega})|_{\boldsymbol{\psi}\equiv 0}\, dt \wedge d{\rm vol}_3$$

$$= \int\limits_{t_1}^{t_2} \left[\int\limits_v \frac{\boldsymbol{J}(\boldsymbol{\omega}, \boldsymbol{\omega})}{2}\, d{\rm vol}_3 \right] \wedge dt = \int\limits_{t_1}^{t_2} \boldsymbol{K}_{\rm rot}\, dt = c\mathfrak{s}_{\rm rot}, \tag{13}$$

where the equality $dt \wedge d{\rm vol}_3 = -d{\rm vol}_3 \wedge dt$ is taken into account, the identification

$$\zeta = \rho c^2 = \varrho c^2 \tag{14}$$

is made and definition of inertia tensor of rigid body:

$$2\boldsymbol{K}_{\rm rot} = \boldsymbol{J} \equiv \int\limits_v \rho(\boldsymbol{x}^2(\ ,\ )_3 - \boldsymbol{x} \otimes \boldsymbol{x})\, d{\rm vol}_3 \tag{15}$$

is used.

Influence of an external volume twist 4-force can be accounted by the 4-density of potential energy $\Upsilon(X)$. Corresponding addendum to the free energy will take the form:

$$\mathfrak{F}_{\text{ext}} = \int\limits_{V} \Upsilon\, dt \wedge d\text{vol}_3 = -\int\limits_{t_1}^{t_2} U\, dt, \tag{16}$$

where

$$U \equiv \int\limits_{v} \Upsilon\, d\text{vol}_3 \tag{17}$$

— 3D potential energy.

6 WEAK BENDING OF A BAR (STRING)

Within standard elasticity theory in case of a weak bending and stretch-contracting the superposition principle takes place: elastic energies for this kinds of strain are calculated independently and then they are combined in a full variational functional. When motion of solids in classical mechanics is considered from the viewpoint of 4D elasticity, one should take into account the both kinds simultaneously, in order to provide accordance with experiment. Really, it is well known, that equations of bars bending have a forth order [8, p.110], while equations of rigid motion of solids — second one. Note, that stretch-contracting elastic energy, in case of a small displacement depends on generalized coordinates derivatives quadratically and provides second order equations of motion of 3D-bodies. However, it gives, in fact, trivial 3D dynamics, since 4-bar under this pure stretch-contracting strains remains non-curved, while general accelerated motion of a 3-body implies curving of its world tube. These arguments lead us to the consideration of a *weak bending of tense bar*. Here and below we consider tension with its sign. Let's introduce more rigorous notions and definitions, generalizing standard bars bending theory to 4D case.

6.1 Relativistic kinematics of a weakly bent bar

Let's pose a bar in its unstrained state in the same coordinate system, that has been taken in previous section and lets consider it from the view point of the same reference frame $\tilde{\boldsymbol{T}}$ (see Fig.3). General strain of the thin bar can be described by orientation of orthonormal tetrad, rigidly connected to points of the bar medium. We pose its origin at the inertia line of the bar $\boldsymbol{X}_0(t) = \{t, \boldsymbol{x}_0(t)\})$ in every 3D section $t = const$ after strain.

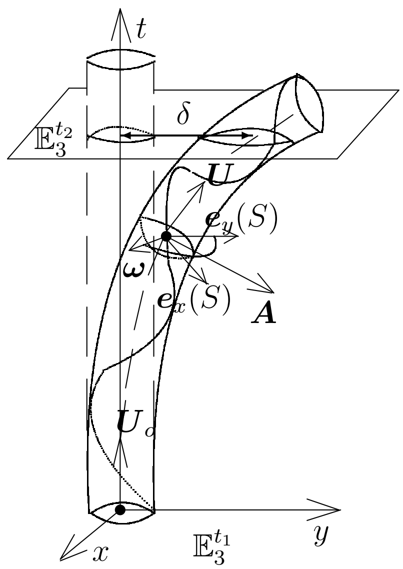

Figure 3: Pure 4-bending of a bar. In unstrained state (shown by dash line) 3-body is in rest. After straining 3-body is twisted and accelerated. Twist vector $\boldsymbol{\omega}$, 4-acceleration $\boldsymbol{A}$ and space-like triad $\boldsymbol{e}$ — are coplanar. $\boldsymbol{U}$ — tangent to inertia line 4-velocity vector, δ — space-like bending deflectio.

In case of a pure bending (without global stretch-contracting) every 3-body, defined by the reference frame $\tilde{\boldsymbol{T}}$, undergoes 4-rotation in $\mathbb{M}_4$, which smoothly depends on length parameter S along the bar. Such rotation can be described by the Lorentzian matrix $\mathrm{L}(S) \in \mathrm{SO}(1,3)$, depending on six parameters: three of them $\{\theta(S)\}$, describe pseudorotations (boosts), and three ones $\{\varphi(S)\}$ — ordinary 3D rotations. Matrix L satisfies the following matrix relation:

$$\mathrm{L}^{\mathrm{T}}\boldsymbol{G}\mathrm{L} = \boldsymbol{G}, \tag{18}$$

where $\boldsymbol{G}$ — pseudoeuclidian metrics, expressing conservation of 4-vector norm under Lorentzian transformations. Form of inertia line of the strained 4-bar can be specified by unit vector field $\boldsymbol{U} \equiv d\boldsymbol{X}_0/dS = \dot{\boldsymbol{X}}_0$, which has physical sense of *4-velocity of 3-body*. If $\boldsymbol{U}_0 \equiv \boldsymbol{U}(0)$ is vector, taken at some (arbitrary but fixed) initial location then for arbitrary S we have:

$$\boldsymbol{U}(S) = \mathrm{L}(S)\boldsymbol{U}_0.$$

Lets consider vector field $\boldsymbol{U}$ at close point $S + dS$ of inertia line:

$$\boldsymbol{U}(S + dS) \approx \boldsymbol{U}(S) + \dot{\boldsymbol{U}}dS = \mathrm{L}(S + dS)\boldsymbol{U}_0 \approx (\mathrm{L}(S) + \dot{\mathrm{L}}dS)\boldsymbol{U}_0,$$

and then

$$\dot{\boldsymbol{U}} \equiv \boldsymbol{A} = \dot{\mathrm{L}}(S)\boldsymbol{U}_0 = \dot{\mathrm{L}}\mathrm{L}^{-1}\boldsymbol{U}(S), \tag{19}$$

where *acceleration 4-vector* $\boldsymbol{A}$ is introduced. Linear operator $\dot{\mathrm{L}}\mathrm{L}^{-1} \equiv \boldsymbol{\Omega}$ can be called *torsion 4-tensor* of the bar. It has a geometrical sense of linear connection: (19) can be rewritten in the form $D\boldsymbol{U} = 0$, where covariant derivative along inertia line operator is defined by relation: $D \equiv d - \boldsymbol{\Omega}(\cdot\ ,\ )dS$.

4-torsion $\boldsymbol{\Omega}$, as it follows directly from its definition (19), is linear form over derivatives $\{\dot{\theta}, \dot{\varphi}\}$. Since $\{\dot{\theta}\} \sim \boldsymbol{A}$, space-like vector $\boldsymbol{A}$ is responsible for bending of the 4D-bar. More exactly, value $|\boldsymbol{A}|^2 = -K^2$, where K — first curvature of the inertia line. Then, weakness of a bending can be formulated through the following Lorentz invariant integral relation:

$$\int\limits_{S_1}^{S_2} K(S)dS \ll 1 \quad \forall\ S_1 < S_2. \tag{20}$$

As it is well known from differential geometry [15], in neighborhood of any point of smooth regular curve the difference between the curve and tangent plane, defined by the pair $\boldsymbol{U}$ and $\boldsymbol{A}$ is value $o(S^2)$, where S — displacement of the point along curve. Consequently all high curvatures can be neglected in calculation of local strain in linear elasticity theory.

The derivatives $\{\dot{\varphi}\}$ are proportional to the angular velocity vector $\boldsymbol{\omega}$, whose relativistic representation has the form:

$$\boldsymbol{\omega} \equiv *(\boldsymbol{\Omega} \wedge \boldsymbol{U}) = \frac{1}{3!}\boldsymbol{E}(\ ,\boldsymbol{\Omega},\boldsymbol{U}) = \frac{1}{3!}\boldsymbol{\epsilon}(\ ,\boldsymbol{\Omega}) \equiv \star\boldsymbol{\Omega}, \tag{21}$$

where $\boldsymbol{\epsilon} \equiv \boldsymbol{E}(\ ,\ ,\ ,\boldsymbol{U}) = *\boldsymbol{U}$ — *Levi-Civita 3-tensor* in $\mathbb{V}_3$, "$\star$" — dual conjugation in $\mathbb{V}_3$. Let us remind that in case of ordinary bar when it is generally bent, the bending strain is inevitably followed by twist one, considered in previous section.

Easy to see that the torsion 4-tensor $\boldsymbol{\Omega}$ is relativistic 4-bar strain measure of a 4-bar and its components and first derivatives (see (11)) are linearly connected with strain tensor $\boldsymbol{D}$. Then quadratic over $\boldsymbol{D}$ expression for a free energy (8) can be written then in terms of quadratic combinations of $\boldsymbol{\Omega}$ and $\dot{\boldsymbol{\Omega}}$:

$$F = \frac{1}{2}\mathcal{J}^{(I)}(\boldsymbol{\Omega},\boldsymbol{\Omega}) + \frac{1}{2}\mathcal{J}^{(II)}(\dot{\boldsymbol{\Omega}},\dot{\boldsymbol{\Omega}}) + \mathcal{J}^{(III)}(\dot{\boldsymbol{\Omega}},\boldsymbol{\Omega}), \tag{22}$$

where it is natural to call functionals $\mathcal{J}^{I,II,III}$ *inertia 4-tensors*. For our purposes, only the first term should be held, since every time derivative implicitly contains multiplier $1/c$, and, consequently, the second and third terms can be related to relativistic corrections. Let $\boldsymbol{\Omega} = \boldsymbol{\Omega}_1(\boldsymbol{A}) + \boldsymbol{\Omega}_2(\boldsymbol{\omega})$, where $\boldsymbol{\Omega}_{1,2}$ — are corresponding linear functionals. After substitution $\boldsymbol{\Omega}$ into (22), we get F as the following quadratic form:

$$F = \frac{1}{2}\mathcal{J}^I_1(\boldsymbol{A},\boldsymbol{A}) + \mathcal{J}^I_2(\boldsymbol{A},\boldsymbol{\omega}) + \frac{1}{2}\mathcal{J}^I_3(\boldsymbol{\omega},\boldsymbol{\omega}). \tag{23}$$

Third term, when strain of 3-body is omitted, must be identical with expression (12) for a pure twist under $\boldsymbol{\psi} = 0$, so $\mathcal{J}_3^I$ is 3-density of inertia tensor $\boldsymbol{J}$ in (15). Second term should be put zero by a time symmetry of elastic energy. In classical language $\mathcal{J}_2^I \equiv 0$ by T-invariance of its laws.

6.2 Apparent kind of the functional $\mathcal{J}_1^I$

For making calculations short, we'll generalize known 3D result for $\mathcal{J}_1^I$ functional [8, p.93]. Let ordinary bar under external influence has curvature in plane (z, x), where z coincides with axe of the bar in undeformed state. Then 3-density of free energy has the form:

$$F = \frac{Ek^2x^2}{2}, \tag{24}$$

where E — Young modulus, relating with Lame coefficients in 3D case by the expression:

$$E = 3\zeta - \frac{\zeta^2}{\lambda + \zeta}, \tag{25}$$

k — curvature of a middle line of the bar at the point. Note, that $k^2 = |\boldsymbol{k}|^2$, where $\boldsymbol{k} = d\boldsymbol{\tau}/dl$ — curvature vector, $\boldsymbol{\tau}$ — unit tangent to inertia line of the bar vector, dl — line element of the bar. In case of general position of the plane of curving, expression (24) can be generalized by the following way:

$$F = \frac{E}{2}(\boldsymbol{x} \otimes \boldsymbol{x})(\boldsymbol{k}, \boldsymbol{k}), \tag{26}$$

Elastic energy per length unit is

$$\frac{d\mathfrak{F}}{dl} = \frac{E}{2}\boldsymbol{i}(\boldsymbol{k}, \boldsymbol{k}),$$

where

$$\boldsymbol{i} \equiv \int_s (\boldsymbol{x} \otimes \boldsymbol{x})$$

— *inertia moment of the section of a bar* and integration is carried out over the cross section of the bar. Full bending energy is

$$\mathfrak{F} = \int_0^l \frac{d\mathfrak{F}}{dl} dl.$$

For generalization to the case of 4-bars we take into account, that for isotropic elastic body in $\mathbb{M}_4$ ([11, 12]):

$$E = \frac{8}{3}\zeta - \frac{4}{9}\frac{\zeta^2}{(\lambda + 2\zeta/3)}, \tag{27}$$

then note, that role of $\boldsymbol{k}$ plays 4-acceleration vector $\boldsymbol{A}$. Finally we generalize expression for $\boldsymbol{i}$:

$$\boldsymbol{I} = \int\limits_{v^t} (\boldsymbol{X} \otimes \boldsymbol{X})\, d\mathrm{vol}_3^t.$$

So, we get

$$\mathcal{J}_1^I = \frac{E}{2}\boldsymbol{I}$$

and finally

$$\mathfrak{F}_b = \int\limits_{S_1}^{S_2} \mathcal{J}_1(\boldsymbol{A}, \boldsymbol{A})\, dt \wedge d\mathrm{vol}_3^t. \tag{28}$$

In the following part of the paper expression (28) will be used only for physical and geometrical restrictions analysis. Really, the part (28) of a full free energy, include second derivatives of displacement vector and so, as it has been mentioned, equilibrium equations will have forth order. In the next subsection we'll construct "right" variational functional $\mathfrak{F}_\mathrm{s}$, which quadratically depends on the first derivatives of displacement vector components and includes tension. For our purposes the following (now obvious) fact is important: *since whole classical mechanics should be contained in $\mathfrak{F}_s$, we have the non-equality*

$$\mathfrak{F}_b \ll \mathfrak{F}_s, \tag{29}$$

which, in fact, redirects our consideration from a general 4-bars physics to the more particular case of 4-strings.

6.3 Bend energy of 4-string

Let's consider 4-string in equilibrium under influence of the forces $\boldsymbol{T}_1 = -\boldsymbol{T}_2$, applied at the ends v^{t_1} and v^{t_2}. This kind of strain is called *simple stretching.*

In ordinary bar deformation theory the following *Saint-Venant* principle is commonly used: [24, p.33]: *external forces distribution at the ends of a bar directly influences on a stress picture only near the ends at the distance about thickness of a bar.* In other words, far from ends, stress tensor depends only on integral values of forces (or its momenta), applied at the ends, but doesn't depend on their local distributions there. Now we postulate validity of the principle in case of 4-strings, defering its physical sense discussion to the Conclusion.

In the simple stretch case the only non-zero component of stress tensor $\boldsymbol{\sigma}$ is

$$\boldsymbol{\sigma}_{tt}(t) = T/\nu_t \equiv \Pi = E\boldsymbol{D}_{tt}, \tag{30}$$

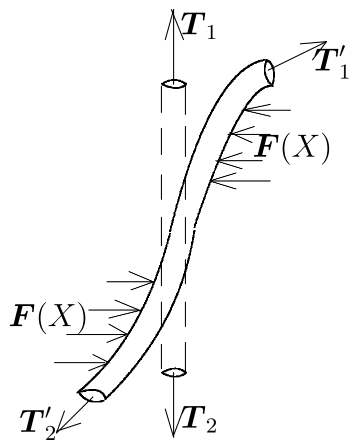

Figure 4: Bending of a tense string. Arrows show bending and stretching forces. Bent string get extrastretching, which energy much more then purely bending one.

where $\boldsymbol{D}_{tt}$ — relative time-like stretching of the string. Expression (30) can be viewed as definition of a Young modulus E. Similarly, the expression for cross strain tensor components can be considered as definition of *Poisson coefficient* σ:

$$\boldsymbol{D}_{xx} = \boldsymbol{D}_{yy} = \boldsymbol{D}_{zz} = -\sigma \boldsymbol{D}_{tt}, \tag{31}$$

where for bodies in $\mathbb{M}_4$ $\sigma = \lambda/(3\lambda + 2\zeta)$ ([11, 12]). Now if the stretched string is affected to bending, then, generally speaking, its integral length will also vary: $S_0 = |t_2 - t_1| \ \to \ S$. Change of the string elastic tension energy under the bending is:

$$F_s = \frac{\boldsymbol{\sigma}_{tt}\boldsymbol{D}_{tt}}{2} = \frac{\Pi}{2}\frac{d\Delta S}{dt}, \tag{32}$$

For the $S - S_0$, taking into account weakness of the bend[9] we have:

$$\Delta S = \int\limits_{S_1}^{S_2} \sqrt{(\boldsymbol{U},\boldsymbol{U})}dS - S_0 = \int\limits_{t_1}^{t_2} \sqrt{1-\boldsymbol{v}^2}\, dt - S_0 \approx -\frac{1}{2}\int\limits_{t_1}^{t_2} \boldsymbol{v}^2\, dt$$

and finally, for stretch elastic energy density

$$F_{\mathrm{s}} = -\frac{\Pi}{4}\boldsymbol{v}^2.$$

Integrating it over 4-volume of the string we get:

$$\mathfrak{F}_{\mathrm{s}} = c\int\limits_{t_1}^{t_2} \frac{m\boldsymbol{v}^2}{2}\, dt = c\mathfrak{s}_{\mathrm{pr}}, \tag{33}$$

[9] Rigorously speaking, we use the *smallness of an inclination* of the 4-string relative to time coordinate line in some reference frame $\tilde{\boldsymbol{T}}$, that can be expressed by the non-equalities $\boldsymbol{v}^2 \ll 1$ at whole length of the 4-string. This condition is, obviously, sufficient, but not necessary for weak bend (20). In ordinary language of velocities and accelerations, we consider nonrelativistic approximation, which doesn't follow from a acceleration smallness of the 3-body.

— classical progressive part of action for a solid, when

$$\frac{\Pi}{2} = \rho c^2 \text{ and } mc^2 = \frac{1}{2}\int \Pi \, d\mathrm{vol}_3^t. \tag{34}$$

Comparing it with (14), we find

$$\zeta = \frac{\Pi}{2} \tag{35}$$

— another "tuning relation" for elastic properties of the 4-string. Potential progressive term can be introduced similarly to the section 5 (formulaes (16)-(17).

At the end of the section lets clear those conditions, which provide identity of the bar deformation theory and ordinary classical mechanics. For this purpose we estimate energies F_b and F_s. For the former, using (28), we have

$$F_b \sim E\mathcal{H}^2 |\boldsymbol{A}|^2 \sim \frac{E\mathcal{H}^2\delta^2}{\mathcal{T}^4},$$

for the latter from the (33):

$$F_s \sim \Pi \left(\frac{\delta}{\mathcal{T}}\right)^2 \sim \frac{T\delta^2}{\mathcal{H}^3\mathcal{T}^2},$$

where $\delta = \max_t \int_{t_1}^t \sqrt{-\boldsymbol{H}(d\boldsymbol{X}_0, d\boldsymbol{X}_0)}$ — *bending deflectio* of 4-string, equal to maximal space-like displacement of 3-body during the time from t_1 to t_2 (see Fig.3).

Remembering condition (29), we get

$$\frac{F_s}{F_b} \sim \frac{T}{E\mathcal{H}^3} \cdot \left(\frac{\mathcal{T}}{\mathcal{H}}\right)^2 \gg 1. \tag{36}$$

Second multiplier in (36) is much more than unit by definition of a thin bar (4) in §3. It means, that the first one $T/E\mathcal{H}^3$ can not be much less then unit. By order of magnitude the denominator is the 4-force T_{cr}, which should be applied to the string to change its length by value about original one, assuming that Hooks law remains valid. So the possibility of pure bending energy omitting in comparison with stretching one for 4-strings (and consequently, validity of classical Newton mechanics), lies in the fact, that *tension of a 4-string is not substantially different from* T_{cr}.

7 CONCLUSION

Let's summarize and discuss obtained results and another physical and mathematical analogies of classical mechanics with multidimensional generalization of elasticity and material strength theories.

1. In standard approach within SR we deal with world of events $\mathbb{M}_4$ and world tubes of bodies, commonly used as convenient way for visualizing of their motion. In the proposed approach the world tube of 3-body is treated as *unified physical object,* existing as 4-body (thin string) and possessing 4D elastic properties[10]. Observer should understand it as *absolute (or objective) history of a some 3-body.* This 3-body is defined by the observers reference frame and its absolute history transforms into *relative (or subjective)* one, following to observer's perception (devices) motion from one simultaneousity hypersurface to another.

2. Previous investigation allow us to write the following relation between elasticity theory of strings and classical mechanics of solids:

$$\mathfrak{F}_{\rm str} = \mathfrak{F}_{\rm str\ s} + \mathfrak{F}_{\rm str\ tw} = c\mathfrak{s}_{\rm pr} + c\mathfrak{s}_{\rm rot}, \tag{37}$$

   where $\mathfrak{F}_{\rm str\ s}$, $\mathfrak{F}_{\rm str\ tw}$ — elastic 4-energies of string extrastretching (induced by its bending) and twist and $\mathfrak{s}_{\rm pr}$, $\mathfrak{s}_{\rm rot}$ — actions for solids progressive and rotational motions correspondingly. This expression is in straightforward analogy to (1) and (2). We list the assumptions to be accepted to obtain (37) in the Table 1. The 1,2,5-th lines are quite obvious. 4-th line restricts general theory of bars deformation to strings one, where any 3D section of a string is managed by ordinary Newton equations. Acceleration dependence of the classical action can appear as a small correction, which can be connected to with modern investigation on modified Newton dynamics (MOND) [25]. The lines 3-rd and 6-th of the table can be treated as some kind of "tuning": elastic properties of a string with respect to twist and stretching-contracting are consistent, so that only one observable parameter — mass 3-density ρ appears in equations. From a general point of view, twist stiffness (rotational inertia) and bending one (progressive inertia) could be independent and quite different. Combining 3-th and 6-th lines and using (27) and (36) after some transformations we get the following relation between 4D elastic modulus: $\lambda/\zeta \sim 1$, that is true for ordinary elastic 3-bodies.

3. Important and interesting circumstance, revealed in the approach is that the only postulates about forces and third Newton law validity are necessary, which allow us to write 4D static equilibrium equations. Mechanical motion and all involved notions have secondary nature: velocity and acceleration have purely

[10] This is impossible in events space of classical mechanics $\mathbb{R} \times \mathbb{E}^3$, since here we have two independent and incompatible metrics — euclidian one and absolute form of time. So world tubes of 3-bodies in $\mathbb{R} \times \mathbb{E}^3$ are not physical, but purely formal objects.

differentially-geometric origin and mass has purely force-like one. Really, classical action include the only integral parameter, which is "response from the ends" — mass density of the 3-body. But according to (34) [11], $mc^2 = \mathrm{T}/2$. Classical law of a mass conservation ($\dot{m} = 0$ in (3)) is then the straightforward consequence of its force nature, since $m(t_1) = m(t_2)$ means $T(t_1) = T(t_2)$ — equilibrium condition of a string along t-direction.

4. Forces, distributed on time-like surface of a string are ordinary contact forces, which we deal with in classical mechanics. By analogy with ordinary 3-strings, which satisfy Saint-Venant principle, its deformation is independent on forces distributions far from the points of its application. The deformation only depends on integral force distribution characteristics: total force $\boldsymbol{F} = \int d\boldsymbol{F}$ defines final bending angle of the 3-string and total force momentum $\boldsymbol{M} = \int \boldsymbol{x} \wedge d\boldsymbol{F}$ — defines value of a constant twist of the 3-string (see.[8, p.91,p.103]). This fact, generalized to 4-strings physics becomes Galilean inertia principle: *strongly tensed 4-string, at regions where there is no bending or twisting forces, remains rectilinear and possesses constant twist.*

5. It is easy to note, that second Newton law, say, for progressive motion expresses static balance of external bending forces and string elastic reaction to the bending equal to $-m\ddot{\boldsymbol{x}}$ in terms of the approach. Consequently, *inertia forces* acquire the sense of elastic reaction forces to bending (progressive inertia) and to twist (rotational inertia.)

   Let's show, that second Newton law can be treated as the special (1D) case of *Laplace formulae*:

$$\Delta p = q\bar{k}\alpha, \tag{38}$$

   which connects under equilibrium normal pressure difference between each side of elastic 2D-membrane with its local surface tension α and local curvature arithmetical mean $\bar{k} = (k_1 + k_2)/2$, where $k_i = 1/R_i$ — main curvatures of the membrane. Geometrical dimensionless factor $q = 2$ for the case of 2D membrane in 3D space. If we change in (38) bending pressure $p \to \boldsymbol{f}$ by the following correspondence

$$d\boldsymbol{f}_{\mathrm{b}} = p\, d\boldsymbol{s} \;\longrightarrow\; d\boldsymbol{F}_{\mathrm{b}} = \boldsymbol{f} dt,$$

   where $\boldsymbol{f}_{\mathrm{b}}, \boldsymbol{F}_{\mathrm{b}}$ — 3D and 4D bending forces, $\boldsymbol{f}$ — 3D ordinary one, $d\boldsymbol{s}$ — normal element of 2D membrane, then $\alpha \to T$ by the correspondence

$$du = \alpha ds \;\longrightarrow\; dU = Tdt = 2mc^2 dt,$$

[11] 4-force has physical dimension of 3-energy.

where du, dU — 2D and 4D elastic potential energy variations correspondingly and finally $\bar{k} \to \boldsymbol{K} = \boldsymbol{A} \approx \boldsymbol{a}/c^2$, then we get from (38) $\boldsymbol{f} = m\boldsymbol{a}$ under $q = 1/2$.

Another useful analogy, supporting our interpreting of mechanics is standard string oscillation problem. The oscillation equation: $\rho u_{tt} = T_{\text{sp}} u_{xx}$, where T_{sp} — space-like tension takes fully symmetric form: $T_{\text{t}} u_{\tau\tau} = T_{\text{sp}} u_{xx}$, if one change $t \to \sqrt{2}ct = \tau$, $2\rho c^2 = T_{\text{t}}$.

6. Boundary conditions at the space-like ends of the string are unavailable for observer, since they lie in causally non-connected with him region of $\mathbb{M}_4$. The Saint-Venant principle generalization, used in above deducing bend energy, in ordinary language means, that information about the ends state is available only for times $\tau \sim \mathcal{H}/c$ near beginning and end of the string. It is the time, that is required for disturbance propagation from the ends along light corners from the past and future ends inside 4-string. For a 3D observer, which could be able to reach this ends regions[12], they would look like some involved and anomalous evolution of 3-body, including its non-stationary rotation and space-like deformation. Inversely, at regions of the string remoted from its ends, 3-body has a regime of "simple motion", which is described by classical Newtons laws.

7. Continuous evolution of 3-bodies in observable physical world means, that there are no isolated 4-strings: each of them has a beginning of its own absolute history and its end (see Fig.1), where corresponding 3-body is naturally formed (or artificially made or destroyed) from another mother 3-bodies or splits into another daughter 3-bodies. These last form in 4D world its own 4-strings too, which have force (non-causal from the SR viewpoint) interaction with middle string (or strings) through the space-like ends. So, the best object, which represents the 4D mechanical structure of our Universe from the viewpoint of present approach, is *space-time tense net.* 3-body Universe of classical mechanics is its simultaneousity section. Similarly, bounded configurations of 3-bodies (for instance planetary system or galaxy clusters) form in Minkowski space-time *4D tense ropes.*

8. From expressions (13) and (33) it is easy to show, that classical action (more precisely $c\mathfrak{s}$ in ordinary units) within elasticity theory has the sense of *4D free strain energy*, that clears its importance in variational formulating of classical mechanics and field theory. We see, that Lagrange function within the former

[12] It may not necessary to be an absolute beginning or end of history of 3D body, but also those moments of time, when the 3D body is essentially influenced by another ones, for example the moment of destroying under non-elastic collision.

appears as its linear 1-density, and lagrangian within the latter — as a volume 4-density. Standard variational mechanical problem with fixed initial and final points of trajectory corresponds to a *4-string with supported ends*, that is one of variety of boundary conditions, appearing within bar deformation theory. The question about true boundary condition of 4-bars and 4-plates should be answered by experimental way (see [14]).

9. Theoretical generalization of classical mechanics within the frame of the approach is possible in two directions. The first one takes into account relativistic corrections (for example expressions (12) in §5) for elastic energy of twist and bending and then compares its influence on shape of the 4-string (in $\mathbb{V}_3^t$ — motion law) with experiment. Second one includes non-linear over strain tensor addenda into free energy, which concern with invalidity of Hooks law, and fixes values of new elastic constants by experiment. Note, that in this non-linear elasticity theory superposition principle, mentioned in the beginning of §6, is, generally speaking, violated.

10. The role of an observer in the approach demands the particular discussion. As a matter of fact, the whole dynamics (i.e. variability of the 3D world) retranslates here into a motion *of perception of an observer from one simultaneousity hypersurface to another.* Such motion has already been postulated in various multidimensional theories ([2]-[17], [27] and ref-s therein). The essence of the postulate in our approach is that *without observer and its perception motion, there is no 3D dynamics — relative history of a mechanical system of 3-bodies but only its absolute one.* So, the concept of observer is necessary not only within the frame of quantum mechanics for resolution of some paradoxes [26, 27], but also in classical physics for formulating of observable laws of physical world.

We believe, that our approach can be useful, at least, from the two points of view. On the one hand, it clears simple and, at the same time, deep 4D origin of laws of classical mechanics and gives obvious ways of its generalization. On the other hand, it also clears general origin of kinetic parts of our lagrangians, while more often physicists hint on potential ones. Using the 4D elastic representation, one can say, that *there are no proper kinetic terms in classical mechanics: they also have a sense of twist and bend potential energies of 4-string, when 4D Hooks law takes place.* Note, that 4-energy density Υ in (16) can be also treated in term of 4D elasticity theory: we only should consider 4-strings as *linear stresses at 4D plate*, which interact with each other by short range forces, described by stress tensor at

the plate. In this picture the plate (or space-time itself) appears as elastic medium with interesting and unusual properties ([11, 12, 13]).

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Table 1. Assumptions, revealing the properties of 4-D elastic thin bars relevant to observable ordinary 3D rigid bodies in classical mechanics.

N	4-elasticity language	mechanical language
1.	Flat sections hypothesis ($\boldsymbol{\psi} = 0$)	Absolutely rigid 3-body
2.	Weak twist ($\boldsymbol{\omega}\mathcal{H} \ll 1$)	Nonrelativistic rotation
3.	$\zeta = \rho c^2$	Elastic nature of rotational inertia
4.	Bar is tense string ($F_b \ll F_s$)	second order of Newton equations
5.	Weak bending (extrastretching) ($\lvert v \rvert \ll 1$)	nonrelativistic progressive motion
6.	$\rho c^2 = \Pi/2$	t-Force nature of mass